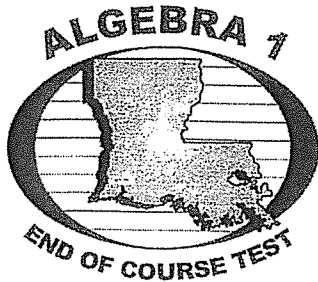


Chapter 4

Solving Equations and Inequalities

This chapter covers the following Algebra 1 Standard(s):



	GLE
Algebra	8

4.1 Two-Step Algebra Problems

In the following two-step algebra problems, **addition** and **subtraction** are performed first and then **multiplication** and **division**.

Example 1: $-4x + 7 = 31$

Step 1: Subtract 7 from both sides.

$$\begin{array}{r} -4x + 7 = 31 \\ -7 \quad -7 \\ \hline -4x = 24 \end{array}$$

Step 2: Divide both sides by -4 .

$$\frac{-4x}{-4} = \frac{24}{-4} \quad \text{so} \quad x = \frac{24}{-4} = -6$$

Example 2: $-8 - y = 12$

Step 1: Add 8 to both sides.

$$\begin{array}{r} -8 - y = 12 \\ +8 \quad +8 \\ \hline -y = 20 \end{array}$$

Step 2: To finish solving a problem with a negative sign in front of the variable, multiply both sides by -1 . The variable needs to be positive in the answer.

$$(-1)(-y) = (-1)(20) \quad \text{so} \quad y = -20$$

Solve the two-step algebra problems below.

- | | | | |
|--------------------|--------------------|---------------------|----------------------|
| 1. $6x - 4 = -34$ | 6. $4x - 12 = 24$ | 11. $-g - 24 = -17$ | 16. $21t + 17 = 80$ |
| 2. $5y - 3 = 32$ | 7. $3x - 17 = -41$ | 12. $-7k - 12 = 30$ | 17. $20y + 9 = 149$ |
| 3. $8 - t = 1$ | 8. $9d - 5 = 49$ | 13. $9 - 5r = 64$ | 18. $15p - 27 = 33$ |
| 4. $10p - 6 = -36$ | 9. $10h + 8 = 78$ | 14. $6y - 14 = 34$ | 19. $22h + 9 = 97$ |
| 5. $11 - 9m = -70$ | 10. $-6b - 8 = 10$ | 15. $12f + 15 = 51$ | 20. $-5 + 36w = 175$ |

4.2 Two-Step Algebra Problems with Fractions

An algebra problem may contain a fraction. Study the following example to understand how to solve algebra problems that contain a fraction.

Example 3: $\frac{x}{2} + 4 = 3$

Step 1:
$$\begin{array}{r} \frac{x}{2} + 4 = 3 \\ -4 \quad -4 \\ \hline \frac{x}{2} = -1 \end{array}$$
 Subtract 4 from both sides.

Step 2: $\frac{x}{2} = -1$ Multiply both sides by 2 to eliminate the fraction.

$$\frac{x}{2} \times 2 = -1 \times 2 \quad \text{so} \quad x = -2$$

Simplify the following algebra problems.

- | | | |
|----------------------------|-----------------------------|------------------------------|
| 1. $4 + \frac{y}{3} = 7$ | 9. $3 + \frac{x}{11} = 7$ | 17. $15 + \frac{z}{14} = 13$ |
| 2. $\frac{a}{2} + 5 = 12$ | 10. $16 + \frac{m}{6} = 14$ | 18. $\frac{b}{6} - 9 = -14$ |
| 3. $\frac{w}{5} - 3 = 6$ | 11. $\frac{p}{3} + 5 = -2$ | 19. $\frac{d}{3} + 7 = 12$ |
| 4. $\frac{x}{9} - 9 = -5$ | 12. $\frac{t}{8} + 9 = 3$ | 20. $10 + \frac{b}{6} = 4$ |
| 5. $\frac{b}{6} + 2 = -4$ | 13. $\frac{v}{7} - 8 = -1$ | 21. $2 + \frac{p}{4} = -6$ |
| 6. $7 + \frac{z}{2} = -13$ | 14. $5 + \frac{h}{10} = 8$ | 22. $\frac{t}{7} - 9 = -5$ |
| 7. $\frac{x}{2} - 7 = 3$ | 15. $\frac{k}{7} - 9 = 1$ | 23. $\frac{a}{10} - 1 = 3$ |
| 8. $\frac{c}{5} + 6 = -2$ | 16. $\frac{y}{4} + 13 = 8$ | 24. $\frac{a}{8} + 16 = 9$ |

4.3 More Two-Step Algebra Problems with Fractions

Study the following example to understand how to solve algebra problems that contain a different type of fraction.

Example 4: $\frac{x+2}{4} = 3$ In this example, " $x+2$ " is divided by 4, and not just the x or the 2.

Step 1: $\frac{x+2}{\cancel{4}} \times \cancel{4} = 3 \times 4$ First multiply both sides by 4 to eliminate the fraction.
 $x+2 = 12$

Step 2: $\begin{array}{r} x+2 = 12 \\ -2 \quad -2 \\ \hline x = 10 \end{array}$ Next, subtract 2 from both sides.

Solve the following problems.

1. $\frac{x+1}{5} = 4$

9. $\frac{13+h}{2} = 12$

17. $\frac{3+t}{3} = 10$

2. $\frac{z-9}{2} = 7$

10. $\frac{k-10}{3} = 9$

18. $\frac{x+5}{5} = -3$

3. $\frac{b-4}{4} = -5$

11. $\frac{a+11}{-4} = 4$

19. $\frac{g+3}{2} = 11$

4. $\frac{y-9}{3} = 7$

12. $\frac{x-20}{7} = 6$

20. $\frac{k+1}{-6} = 5$

5. $\frac{d-10}{-2} = 12$

13. $\frac{t+2}{6} = -5$

21. $\frac{y-14}{2} = -8$

6. $\frac{w-10}{-8} = -4$

14. $\frac{b+1}{-7} = 2$

22. $\frac{z-4}{-2} = 13$

7. $\frac{x-1}{-2} = -5$

15. $\frac{f-9}{3} = 8$

23. $\frac{w+2}{15} = -1$

8. $\frac{c+40}{-5} = -7$

16. $\frac{4+w}{6} = -6$

24. $\frac{3+h}{3} = 6$

4.4 Simplifying/Combining Like Terms

In an algebra problem, separate **terms** by + and - signs. The expression $5x - 4 - 3x + 7$ has 4 terms: $5x$, 4 , $3x$, and 7 . Terms having the same variable can be combined (added or subtracted) to simplify the expression. $5x - 4 - 3x + 7$ simplifies to $2x + 3$.

$$\begin{array}{c}
 \swarrow \quad \searrow \\
 5x - 4 - 3x + 7 \\
 \nwarrow \quad \nearrow \\
 5x - 3x \quad -4 + 7 = 2x + 3
 \end{array}$$

Simplify the following expressions.

- | | | |
|-------------------|------------------------|-----------------------|
| 1. $7x + 12x$ | 8. $21 - 10t + 9 - 2t$ | 15. $-d + 1 + 2d - 4$ |
| 2. $8y - 5y + 8$ | 9. $-3 + x - 4x + 9$ | 16. $-8 + 4h + 1 - h$ |
| 3. $4 - 2x + 9$ | 10. $7b + 12 + 4b$ | 17. $12x - 4x + 7$ |
| 4. $11a - 16 - a$ | 11. $4h - h + 2 - 5$ | 18. $10 + 3z + z - 5$ |
| 5. $9w + 3w + 3$ | 12. $-6k + 10 - 4k$ | 19. $14 + 3y - y - 2$ |
| 6. $-5x + x + 2x$ | 13. $2a + 12a - 5 + a$ | 20. $11p - 4 + p$ |
| 7. $w - 15 + 9w$ | 14. $5 + 9c - 10$ | 21. $11m + 2 - m + 1$ |

4.5 Solving Equations with Like Terms

When an equation has two or more like terms on the same side of the equation, combining like terms is the **first** step in solving the equation.

Example 5: $7x + 2x - 7 = 21 + 8$

Step 1:	Combine like terms on both sides of the equation.	$ \begin{array}{r} 7x + 2x - 7 = 21 + 8 \\ 9x - 7 = 29 \\ +7 \quad +7 \\ 9x \div 9 = 36 \div 9 \\ x = 4 \end{array} $
Step 2:	Solve the two-step algebra problem as explained previously.	

Solve the equations below combining like terms first.

- | | | |
|----------------------------|----------------------------|-----------------------------|
| 1. $3w - 2w + 4 = 6$ | 5. $-2t + 4t - 7 = 9$ | 9. $-4 - 3x - x = -16$ |
| 2. $7x + 3 + x = 16 + 3$ | 6. $9d + d - 3d = 14$ | 10. $9 - 12p + 5p = 14 + 2$ |
| 3. $5 - 6y + 9y = -15 + 5$ | 7. $-6c - 4 - 5c = 10 + 8$ | 11. $10y + 4 - 7y = -17$ |
| 4. $-14 + 7a + 2a = -5$ | 8. $15m - 9 - 6m = 9$ | 12. $-8a - 15 - 4a = 9$ |

4.5 Solving Equations with Like Terms

If the equation has like terms on both sides of the equation, you must get all of the terms with a **variable** on one side of the equation and all of the **integers** on the other side of the equation.

Example 6: $3x + 2 = 6x - 1$

Step 1: Subtract $6x$ from both sides to move all the **variables** to the left side.

Step 2: Subtract 2 from both sides to move all the **integers** to the right side.

Step 3: Divide by -3 to solve for x .

$$\begin{array}{r}
 3x + 2 = 6x - 1 \\
 -6x \quad -6x \\
 \hline
 -3x + 2 = -1 \\
 -2 \quad -2 \\
 \hline
 -3x = -3 \\
 -3 \quad -3 \\
 \hline
 x = 1
 \end{array}$$

Solve the following problems.

1. $3a + 1 = a + 9$

11. $16y + 12 = 9y + 33$

21. $-12x + 14 = 8x - 46$

2. $2d - 12 = d + 3$

12. $13 - 11w = 3 - w$

22. $27 - 11h = 5 - 9h$

3. $5x + 6 = 14 - 3x$

13. $-17b + 23 = -4 - 8b$

23. $5t + 36 = -6 - 2t$

4. $15 - 4y = 2y - 3$

14. $k + 5 = 20 - 2k$

24. $17y + 42 = 10y + 7$

5. $9w - 7 = 12w - 13$

15. $12 + m = 4m + 21$

25. $22x - 24 = 14x - 8$

6. $10b + 19 = 4b - 5$

16. $7p - 30 = p + 6$

26. $p - 1 = 4p + 17$

7. $-7m + 9 = 29 - 2m$

17. $19 - 13z = 9 - 12z$

27. $4d + 14 = 3d - 1$

8. $5x - 26 = 13x - 2$

18. $8y - 2 = 4y + 22$

28. $7w - 5 = 8w + 12$

9. $19 - p = 3p - 9$

19. $5 + 16w = 6w - 45$

29. $-3y - 2 = 9y + 22$

10. $-7p - 14 = -2p + 11$

20. $-27 - 7x = 2x + 18$

30. $17 - 9m = m - 23$

4.6 Removing Parentheses

The distributive property is used to remove parentheses.

Example 7: $2(a + 6)$

You multiply 2 by each term inside the parentheses. $2 \times a = 2a$ and $2 \times 6 = 12$. The 12 is a positive number so use a plus sign between the terms in the answer.

$$2(a + 6) = 2a + 12$$

Example 8: $4(-5c + 2)$

The first term inside the parentheses could be negative. Multiply in exactly the same way as the examples above. $4 \times (-5c) = -20c$ and $4 \times 2 = 8$

$$4(-5c + 2) = -20c + 8$$

Remove the parentheses in the problems below and simplify the following expressions.

1. $7(n + 6)$

6. $4(d - 8)$

11. $10(9 - y)$

2. $8(2g - 5)$

7. $2(-4x + 6)$

12. $9(c - 9)$

3. $11(5z - 2)$

8. $7(4 + 6p)$

13. $12(-3t + 1)$

4. $6(-y - 4)$

9. $5(-4w - 8)$

14. $3(4y + 9)$

5. $3(-3k + 5)$

10. $6(11x + 2)$

15. $8(b + 3)$

The number in front of the parentheses can also be negative. Remove these parentheses the same way.

Example 9: $-2(b - 4)$

First, multiply $-2 \times b = -2b$

Second, multiply $-2 \times -4 = 8$

Copy the two products. The second product is a positive number so put a plus sign between the terms in the answer.

$$-2(b - 4) = -2b + 8$$

Remove the parentheses in the following problems and simplify the following expressions.

16. $-7(x + 2)$

21. $-3(4x - 2)$

26. $-3(9 - 7p)$

17. $-5(4 - y)$

22. $-2(-z + 2)$

27. $-9(-k - 3)$

18. $-4(2b - 2)$

23. $-4(7p + 7)$

28. $-1(7b - 9)$

19. $-2(8c + 6)$

24. $-9(t - 6)$

29. $-6(-5t - 2)$

20. $-5(-w - 8)$

25. $-10(2w + 4)$

30. $-7(-v + 4)$

4.7 Multi-Step Algebra Problems

You can now use what you know about removing parentheses, combining like terms, and solving simple algebra problems to solve problems that involve three or more steps. Study the examples below to see how easy it is to solve multi-step problems.

Example 10: $3(x + 6) = 5x - 2$

Step 1:	Use the distributive property to remove parentheses.	$3x + 18 = 5x - 2$
Step 2:	Subtract $5x$ from each side to move the terms with variables to the left side of the equation.	$\frac{-5x}{-2x + 18} = \frac{-5x}{-2}$
Step 3:	Subtract 18 from each side to move the integers to the right side of the equation.	$\frac{-18}{-2x} = \frac{-18}{-2}$
Step 4:	Divide both sides by -2 to solve for x .	$x = 10$

Example 11: $\frac{3(x - 3)}{2} = 9$

Step 1:	Use the distributive property to remove parentheses.	$\frac{3x - 9}{2} = 9$
Step 2:	Multiply both sides by 2 to eliminate the fraction.	$\frac{2(3x - 9)}{2} = 2(9)$
Step 3:	Add 9 to both sides, and combine like terms.	$\frac{3x - 9}{+9} = \frac{18}{+9}$
Step 4:	Divide both sides by 3 to solve for x .	$\frac{3x}{3} = \frac{27}{3}$ $x = 9$

Solve the following multi-step algebra problems.

1. $2(y - 3) = 4y + 6$

5. $2x + 3x = 30 - x$

2. $\frac{2(a + 4)}{2} = 12$

6. $\frac{2a + 1}{3} = a + 5$

3. $\frac{10(x - 2)}{5} = 14$

7. $5(b - 4) = 10b + 5$

4. $\frac{12y - 18}{6} = 4y + 3$

8. $-8(y + 4) = 10y + 4$

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$$9. \frac{x+4}{-3} = 6 - x$$

$$10. \frac{4(n+3)}{5} = n - 3$$

$$11. 3(2x - 5) = 8x - 9$$

$$12. 7 - 10a = 9 - 9a$$

$$13. 7 - 5x = 10 - (6x + 7)$$

$$14. 4(x - 3) - x = x - 6$$

$$15. 4a + 4 = 3a - 4$$

$$16. -3(x - 4) + 5 = -2x - 2$$

$$17. 5b - 11 = 13 - b$$

$$18. \frac{-4x+3}{2x} = \frac{7}{2x}$$

$$19. -(x+1) = -2(5-x)$$

$$20. 4(2c+3) - 7 = 13$$

$$21. 6 - 3a = 9 - 2(2a+5)$$

$$22. -5x + 9 = -3x + 11$$

$$23. 3y + 2 - 2y - 5 = 4y + 3$$

$$24. 3y - 10 = 4 - 4y$$

$$25. -(a+3) = -2(2a+1) - 7$$

$$26. 5m - 2(m+1) = m - 10$$

$$27. \frac{1}{2}(b-2) = 5$$

$$28. -3(b-4) = -2b$$

$$29. 4x + 12 = -2(x+3)$$

$$30. \frac{7x+4}{3} = 2x - 1$$

$$31. 9x - 5 = 8x - 7$$

$$32. 7x - 5 = 4x + 10$$

$$33. \frac{4x+8}{2} = 6$$

$$34. 2(c+4) + 8 = 10$$

$$35. y - (y+3) = y + 6$$

$$36. 4 + x - 2(x-6) = 8$$